Network structure in two-winner combinatorial group contest

Anwesha Mukerjee

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Definitions

- A contest is a game where players spend costly resources in order to win valuable rewards and the spent resources are sunk irrespective of the final outcome.

  - Sports tournaments
  - Political election
  - R&D conests
  - Tenders for Government projects

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- A multi-winner contest is a contest where there are more than one winners and any individual player can win at most one reward.

  - Admission into a university course
  - Acceptance of a research paper in a conference
  - Allocation of quotas

Definitions

- A k-winner ($k > 1$) combinatorial group contest is a contest where the possible overlapping k-player groups of winners are defined by given preferences and the members of only one such group receive the k rewards.

  (*a la* Chowdhury and Kovenock, 2012)

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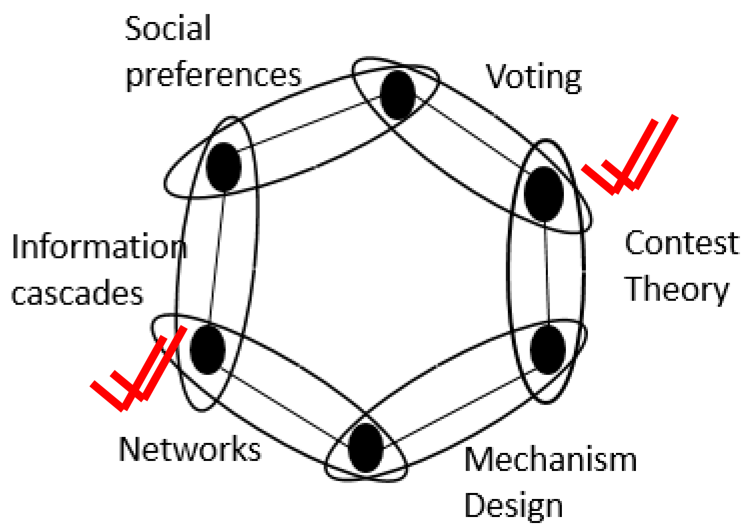

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Examples: Choosing a group of representatives from a larger population.

- Selection of civilians for council membership.
- Selection of a set of employees for working together on a new project.
- Choice of research papers in a conference session
- Photography contests and designing contests.

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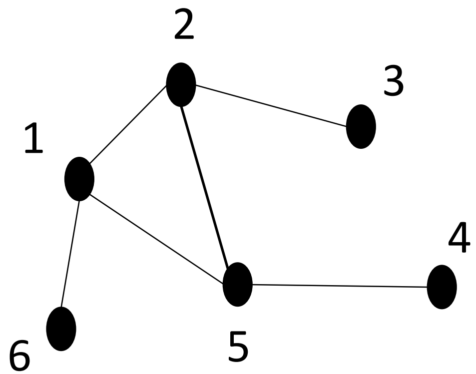

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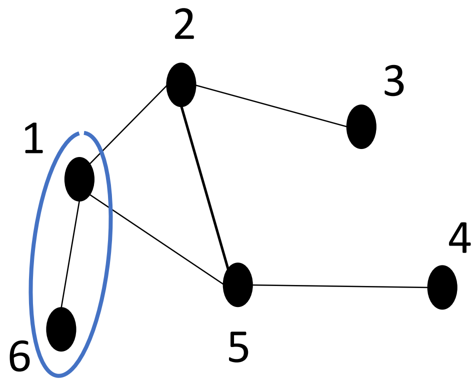

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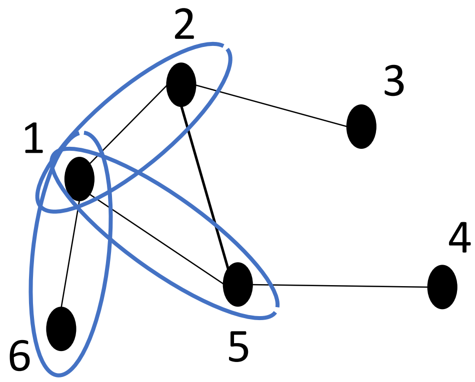

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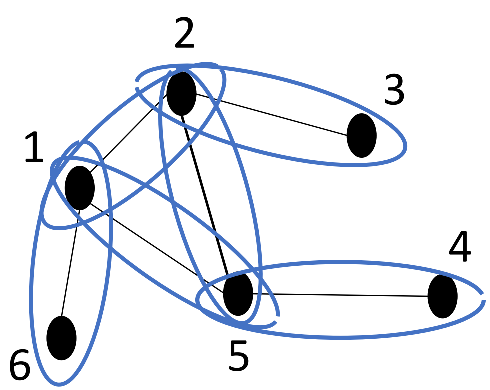

- Six permissible winning coalitions.
- Each player is a part of as many prospective coalitions as the number of his direct neighbours.

Contest success function for $k = 2$

The contest success function for player i is given by

$$p_i = \frac{n_i x_i + \sum_{j \in N_i} x_j}{n_i x_i + \sum_{j \neq i} n_j x_j}$$

Where x_i is the effort input (resource outlay) of player i

N_i is the set of direct neighbours of player i

$n_i = |N_i|$

Equilibrium outlay for $k = 2$

The Expected payoff to player i is given by

$$\pi_i = \frac{n_i x_i + \sum_{j \in N_i} x_j}{n_i x_i + \sum_{j \neq i} n_j x_j} V_i - c_i x_i$$

(where c_i is the marginal effort cost and V_i is the value of a reward to i)

Which player i maximizes under the constraint $x_i \geq 0$

Equilibrium outlay for $k = 2$

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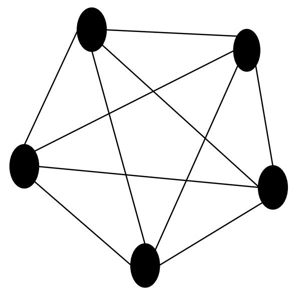
Which player i maximizes under the constraint $x_i \geq 0$

The equilibrium effort is given by

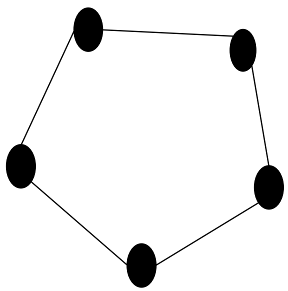
$$x_i^* = max\left\{ 0 , \left(\frac{V_i}{n_i c_i}\right)^{\frac{1}{2}} \left(\sum_{j \in N_i} (n_j - 1) x_j + \sum_{j \in N \setminus N_i \cup \{i\}} n_j x_j \right)^{\frac{1}{2}} - \frac{1}{n_i} \sum_{i \neq j} n_j x_j \right\}$$

Contribution to the literature

Existing Literature

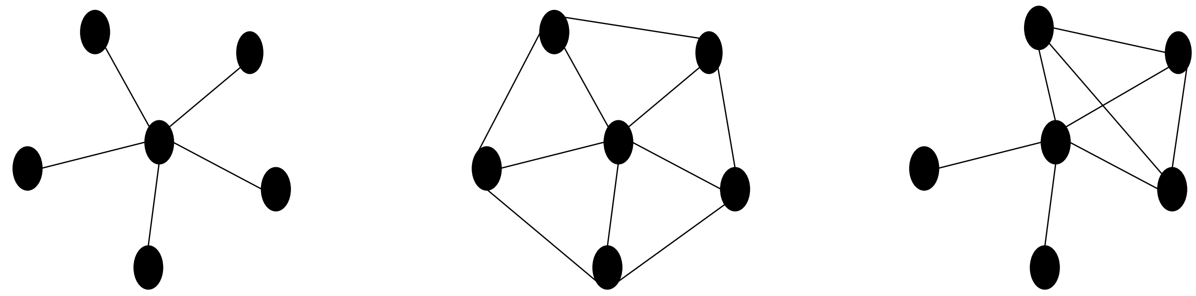


Berry (1993)

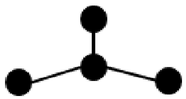
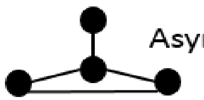
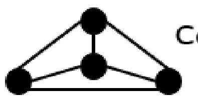
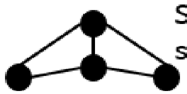
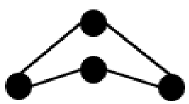
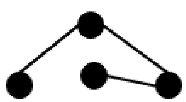


Chowdhury and Kovenock (1993)

Present work



All possible network structures for $N = 4$ and $k = 2$ and corresponding equilibrium payoffs

Network structure with 4 players	Degree 1		Degree 2		Degree 3	
	# players	Payoff	# players	Payoff	# players	Payoff
 Star	3	$v/9$	0	_	1	v
 Asymmetric	1	$v/7$	2	$22v/49$	1	$29v/49$
 Complete	0	_	0	_	4	$3v/8$
 Semi-symmetric	0	_	2	$7v/25$	2	$12v/25$
 Ring	0	_	4	$3v/8$	0	_
 Line	2	$2v/9$	2	$5v/9$	0	_

$$\frac{v}{9} < \frac{v}{7} < \frac{2v}{9} < \frac{7v}{25} < \frac{3v}{8} < \frac{22v}{49} < \frac{12v}{25} < \frac{5v}{9} < \frac{29v}{49} < v$$

Network Stability for $N = 4$ and $k = 2$

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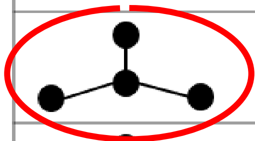
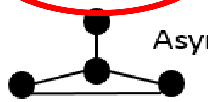
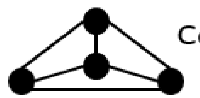
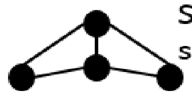
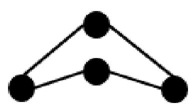
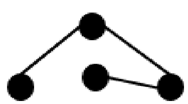
 Star	3	$v/9$	0	_	1	v
 Asymmetric	1	$v/7$	2	$22v/49$	1	$29v/49$
 Complete	0	_	0	_	4	$3v/8$
 Semi-symmetric	0	_	2	$7v/25$	2	$12v/25$
 Ring	0	_	4	$3v/8$	0	_
 Line	2	$2v/9$	2	$5v/9$	0	_

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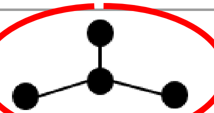
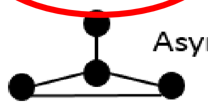
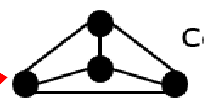
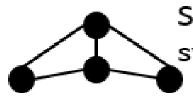
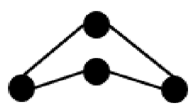
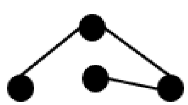
 Star	3	$v/9$	0	_	1	v
 Asymmetric	1	$v/7$	2	$22v/49$	1	$29v/49$
 Complete	0	_	0	_	4	$3v/8$
 Semi-symmetric	0	_	2	$7v/25$	2	$12v/25$
 Ring	0	_	4	$3v/8$	0	_
 Line	2	$2v/9$	2	$5v/9$	0	_

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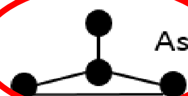
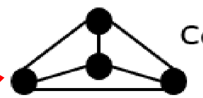
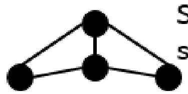
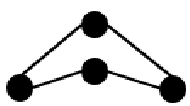
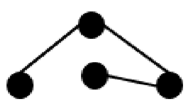
 Asymmetric	1	$v/7$	2	$22v/49$	1	$29v/49$
 Complete	0	_	0	_	4	$3v/8$
 Semi-symmetric	0	_	2	$7v/25$	2	$12v/25$
 Ring	0	_	4	$3v/8$	0	_
 Line	2	$2v/9$	2	$5v/9$	0	_

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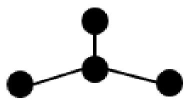
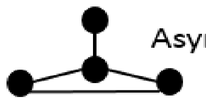
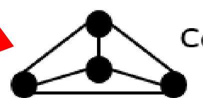
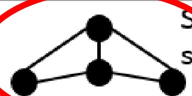
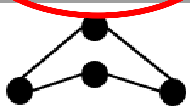
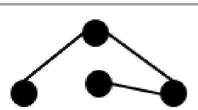
 Star	3	$v/9$	0	_	1	v
 Asymmetric	1	$v/7$	2	$22v/49$	1	$29v/49$
 Complete	0	_	0	_	4	$3v/8$
 Semi-symmetric	0	_	2	$7v/25$	2	$12v/25$
 Ring	0	_	4	$3v/8$	0	_
 Line	2	$2v/9$	2	$5v/9$	0	_

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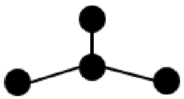
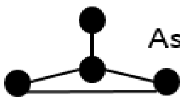
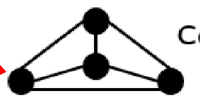
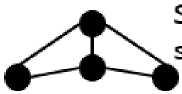
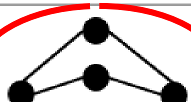
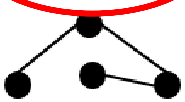
 Star	3	$v/9$	0	_	1	v
 Asymmetric	1	$v/7$	2	$22v/49$	1	$29v/49$
 Complete	0	_	0	_	4	$3v/8$
 Semi-symmetric	0	_	2	$7v/25$	2	$12v/25$
 Ring	0	_	4	$3v/8$	0	_
 Line	2	$2v/9$	2	$5v/9$	0	_

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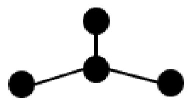
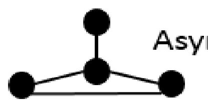
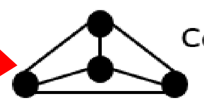
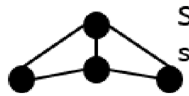
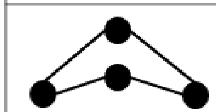
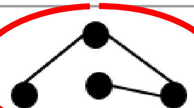
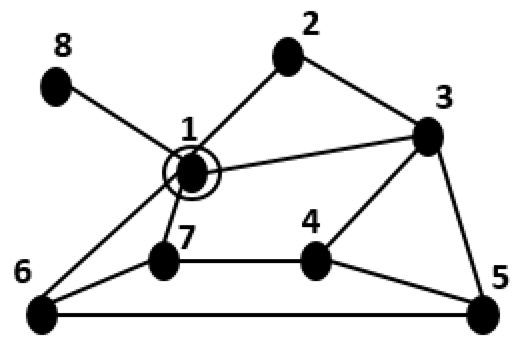
 Star	3	$v/9$	0	_	1	v
 Asymmetric	1	$v/7$	2	$22v/49$	1	$29v/49$
 Complete	0	_	0	_	4	$3v/8$
 Semi-symmetric	0	_	2	$7v/25$	2	$12v/25$
 Ring	0	_	4	$3v/8$	0	_
 Line	2	$2v/9$	2	$5v/9$	0	_

$$\frac{v}{9} < \frac{v}{7} < \frac{2v}{9} < \frac{7v}{25} < \frac{3v}{8} < \frac{22v}{49} < \frac{12v}{25} < \frac{5v}{9} < \frac{29v}{49} < v$$

Contest success function for $k = 3$



Number of permissible 3-partite coalition that a player i is a part of:

$$N_i' = \binom{n_i}{2} + \sum_{t \in N_i} |N_t \backslash N_i \cup \{i\}|$$

Contest success function for $k = 3$

The contest success function for player i is given by

$$P_i = \frac{\binom{n_i}{2} x_i + (n_i - 1)\sum_{t \in N_i} x_t + \sum_{t \in N_i}\left[|N_t \backslash N_i \cup \{i\}|(x_i + x_t) + \sum_{j \in N_t \backslash N_i \cup \{i\}} x_j\right]}{\sum_{j \in N}\left[\binom{n_i}{2} x_j + \sum_{t \in N_j}\left|N_t \backslash N_j \cup \{j\}\right| x_j\right]}$$

Where x_i is the effort input (resource outlay) of player i

N_i is the set of direct neighbours of player i

$n_i = |N_i|$

Equilibrium outlay for $k = 3$

The equilibrium effort is given by

$$x_i^{\ *} = \max\left\{0, \left(\frac{v_i}{N_i^{'}}\right)^{\frac{1}{2}} (T_{-i} - M_{-i})^{\frac{1}{2}} - \frac{T_{-i}}{N_i^{'}}\right\}$$

$Where$

$$M_{-i} = (n_i - 1)\sum_{t \in N_i} x_t + \sum_{t \in N_i}\left[|N_t \backslash N_i \cup \{i\}| x_t + \sum_{j \in N_t \backslash N_i \cup \{i\}} x_j\right]$$

$$T_{-i} = \sum_{j \in N \backslash \{i\}}\left[\binom{n_j}{2} + \sum_{t \in N_j}\left|N_t \backslash N_j \cup \{j\}\right|\right] x_j$$

and

$$N_i^{'} = \binom{n_i}{2} + \sum_{t \in N_i} |N_t \backslash N_i \cup \{i\}|$$

Conclusion

- This study characterizes the equilibrium effort for 2-winner and 3-winner combinatorial contests, and argues that

  - High linkage players receive higher equilibrium payoffs.

    Irregular network → Complete network.

  - The effect of change in degree of a player has dynamic consequences on other players' equilibrium efforts and may lead to further adjustment in the network structure.

- The main contribution of this paper is to generalize the network structure in order to accommodate irregular networks, which however comes at a cost of restricting the number of winners.